STAT 510: Homework 01

David Dalpiaz

Due: Friday, September 04, 11:59 PM

General Directions

This assignment is worth 10 points. For each exercise, you may obtain a score of 0, 0.5, or 1.

- To obtain a score of **1**, your answer must be correct, contain valid supporting work, and be reasonably formatted up to and including boxing your answer when possible.
- A score of **0.5** will be given to solutions which show reasonabe effort, but contain errors. (A score of **1** may be granted to some solutions containing errors if they are extremely minor.)
- A score of **0** will be given to a blank solution or a solution that shows no reasonable progress towards the correct solution. Note that if you do not indicate a page for a problem on Gradescope, it will be considered blank.

Please submit your assignment to Gradescope by the due date listed above. You may submit up to 48 hours late with a two point late penalty. After that, no late work will be accepted.

Any grade disputes must be petitioned through Gradescope within one week of receiving a grade.

Please attempt to submit your work as a single PDF and complete the process of indicating which problem is on which page. You may need to merge together PDF files from various sources and scans. We will keep track of best practice for submitting to Gradescope in this Piazza thread.

Homework assignments are meant to be learning experiences. You may discuss the exercises with other students, but you must write the solutions on your own. Directly sharing or copying any part of a homework solution is an infraction of the University's rules on academic integrity. Any violation will be punished as severely as possible.

For this, and all homework assignments, you may use any computational tools that you wish, such as a statistical computing enviroment or integral solver. The course staff is most familiar with `R`, so we will be able to best support `R` users, but you may use any software that you like.

Practice Exercises

The following exercises from Evans and Rosenthal contain back-of-the-book solutions.

- Section 1.2: 1, 3, 5, 9, 11
- Section 1.3: 1, 3, 5
- Section 1.4: 1
- Section 1.5: 1, 3, 5, 7, 9, 11
- Section 2.1: 5, 9
- Section 2.2: 1, 7
- Section 2.3: 1, 3, 7, 9, 11, 15
- Section 2.4: 1, 3, 5, 7, 9
- Section 2.5: 7, 15
- Section 2.6: 5, 7, 9
- Section 2.7: 1, 3, 7, 9

- Section 2.8: 1, 3, 5, 7, 13
- Section 2.9: 7, 9
- Section 2.10: 1, 3, 5, 7, 13

This is an unusually large number of practice problems. In the future there may be far fewer. You do not *need* to do each of these exercises. However, we expect that you are able to complete each of these exercises based on previous coursework. While exam problems will generally be most similar to graded exercises, some exercises similar to practice exercises in difficulty may appear on the exam to stabalize the variance of scores.

Graded Exercises

Exercise 1 (Independent Events)

Let A and B be independent events. Show that the following are independent:

- A^c and B
- A and B^c
- A^c and B^c

Exercise 2 (Conditional Probability with Cards)

(Based on **LW** 1.12) Suppose we have three cards:

- The first is green on both sides.
- The second is red on both sides.
- The third is green on one side and red on the other.

We choose a card at random and we see one side. (The side we see is also random.) If the side we see is green, what is the probability that the other side is also green?

Many people intuitively answer 1/2. Calculate the correct answer.

Exercise 3 (Bayes' Theorem)

Given:

- $\text{P}(Y = A) = 0.09$
- $\text{P}(Y = B) = 0.18$
- $\text{P}(Y = C) = 0.73$
- $X \mid Y = A \sim \text{Poisson}(\lambda_A = 4.25)$
- $X \mid Y = B \sim \text{Poisson}(\lambda_B = 6.14)$
- $X \mid Y = C \sim \text{Poisson}(\lambda_C = 8.51)$

Calculate $\text{P}(Y = B \mid X = 3)$.

Exercise 4 (Normal Distribution)

(Based on **LW** 2.18) Let $X \sim \text{Normal}(\mu = 2.5, \sigma = 3.2)$.

- Calculate $P(X < 4)$.
- Calculate $P(X > 2)$.
- Find x such that $P(X > x) = 0.05$.
- Calculate $P(X = 4)$.
- Calculate $P(0 < X \leq 3)$.

Exercise 5 (Single Variable Transformation)

(Based on **LW** 2.4) Let X have the probability density function

$$f_X(x) = \begin{cases} 1/4 & 0 < x < 1 \\ 3/8 & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

Define $Y = 1/X$. Find the probability density function of Y.

Exercise 6 (Conditional Distribution)

(Based on **LW** 2.17) Given

$$f_{X,Y}(x, y) = \begin{cases} c(x+y)^2 & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X < 0.5 \mid Y = 0.5)$.

Exercise 7 (Conditional Poissons)

(Based on **LW** 2.16) Let $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$ and assume that X and Y are independent. Find the distribution of X given that $X + Y = n$.

Exercise 8 (Difference Distribution)

Let $X \sim \text{Uniform}(0, 1)$ and $Y \sim \text{Uniform}(0, 1)$ be independent. Find the probability density function of $X - Y$.

Exercise 9 (Ratio Distribution)

Let $X \sim \text{Uniform}(0, 1)$ and $Y \sim \text{Uniform}(0, 1)$ be independent. Find the probability density function of X/Y.

Exercise 10 (Simulation Study)

Use a computer experiment to verify your results to Exercise 8 and Exercise 9. For each, do the following:

- Generate a vector $x = (x_1, x_2, \ldots x_{1000})$ from a Uniform$(0, 1)$ distribution.
- Generate a vector $y = (x_1, x_2, \ldots x_{1000})$ from a Uniform$(0, 1)$ distribution.
- Define $z = x - y$ or $z = x/y$.
- Plot a histogram of z. (Be sure to use density histogram, not frequency.)
- Overlay the true density that you calculated.

You may use any computational tool of your choice. Where possible, please supply your code, and of course, the two histograms. If you are using `R`, we recommend using `breaks = 1000`, `xlim = c(0, 25)`, and `ylim = c(0, 0.55)` for the plot of $z = x/y$. (Otherwise, it may be difficult to see that your density matches the histogram.)

Exercise 11 (BYOQ: Bring Your Own Question)

Submit your own question with a solution! If accepted, you will receive one **buffer point**. To be accepted:

- Your question must be reasonably challenging. It should be at least as challenging as the "average" question on this assignment.

- Is must be **original**. If, based on some quick searching we can find the *exact* question (that is, you just copy-pasted a question), you will **lose** a point instead of receiving a buffer point. If we find that it is simply a derivative of another exercise (for example, just changing a constant) you will not receive a point.

The instructor may create videos solving these, or they may be circulated for additional practice.