STAT 510: Homework 08

David Dalpiaz

Due: Friday, November 13, 11:59 PM

General Directions

This assignment is worth 10 points with the potential to obtain one buffer point. For each exercise, you may obtain a score of 0, 0.5, or 1.

- To obtain a score of **1**, your answer must be correct, contain valid supporting work, and be reasonably formatted up to and including boxing your answer when possible.
- A score of **0.5** will be given to solutions which show reasonabe effort, but contain errors. (A score of **1** may be granted to some solutions containing errors if they are extremely minor.)
- A score of **0** will be given to a blank solution or a solution that shows no reasonable progress towards the correct solution. Note that if you do not indicate a page for a problem on Gradescope, it will be considered blank.

Please submit your assignment to Gradescope by the due date listed above. You may submit up to 48 hours late with a two point late penalty. After that, no late work will be accepted.

Any grade disputes must be petitioned through Gradescope within one week of receiving a grade.

Please attempt to submit your work as a single PDF and complete the process of indicating which problem is on which page. You may need to merge together PDF files from various sources and scans. We will keep track of best practice for submitting to Gradescope in this Piazza thread.

Homework assignments are meant to be learning experiences. You may discuss the exercises with other students, but you must write the solutions on your own. Directly sharing or copying any part of a homework solution is an infraction of the University's rules on academic integrity. Any violation will be punished as severely as possible.

For this, and all homework assignments, you may use any computational tools that you wish, such as a statistical computing enviroment or integral solver. The course staff is most familiar with `R`, so we will be able to best support `R` users, but you may use any software that you like.

Graded Exercises

Exercise 1 (Using a Wald Test)

(**LW** 10.7) In 1861, 10 essays appeared in the *New Orleans Daily Crescent*. They were signed "Quintus Curtius Snodgrass" and some people suspected they were actually written by Mark Twain. To investigate this, we will consider the proportion of three letter words found in an author's work.

From eight Twain essays we have:

```
twain = c(0.225, 0.262, 0.217, 0.240, 0.230, 0.229, 0.235, 0.217)
```

From 10 Snodgrass essays we have:

```
snod = c(0.209, 0.205, 0.196, 0.210, 0.202, 0.207, 0.224, 0.223, 0.220, 0.201)
```

Perform a Wald test for equality of the means. Use the nonparametric plug-in estimator. Report the p-value and an approximate 95 percent confidence interval for the difference of means. What do you conclude?

Exercise 2 (Using a Permutation Test)

(**LW** 10.7) In 1861, 10 essays appeared in the *New Orleans Daily Crescent*. They were signed "Quintus Curtius Snodgrass" and some people suspected they were actually written by Mark Twain. To investigate this, we will consider the proportion of three letter words found in an author's work.

From eight Twain essays we have:

```
twain = c(0.225, 0.262, 0.217, 0.240, 0.230, 0.229, 0.235, 0.217)
```

From 10 Snodgrass essays we have:

```
snod = c(0.209, 0.205, 0.196, 0.210, 0.202, 0.207, 0.224, 0.223, 0.220, 0.201)
```

Perform a permutation test to test for equality of the means. Report the p-value. What is your conclusion?

Exercise 3 (Power and Size)

(**LW** 10.5) Let $X_1, \ldots, X_n \sim \text{Uniform}(0, \theta)$ and define $Y_n = \max\{X_1, \ldots, X_n\}$. We want to test

$$H_0 : \theta = 0.5 \quad \text{versus} \quad H_1 : \theta > 0.5.$$

The Wald test is not appropriate since Y does not converge to a Normal. So, suppose we decide to test this hypothesis by rejecting H_0 when $Y > c$.

- Find the power function.
- What choice of c will make the size of the test 0.05?
- In a sample of size $n = 20$ with $Y = 0.48$ what is the p-value? What conclusion about H_0 would you make?
- In a sample of size $n = 20$ with $Y = 0.52$ what is the p-value? What conclusion about H_0 would you make?

Exercise 4 (Two Simple Hypotheses)

(**LW** 10.8) Let $X_1, \ldots, X_n \sim N(\theta, 1)$. Consider testing

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta = 1.$$

Let the rejection region be $R = \{x^n : T(x^n) > c\}$ where $T(x^n) = \frac{1}{n} \sum_{i=1}^{n} X_i$.

- Find c so that the test has size α.
- Find the power under H_1, that is, find, $\beta(1)$.
- Show that, $\beta(1) \to 1$ as $n \to \infty$.

Exercise 5 (Testing for a Normal Mean)

(**LW** 10.13) Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. Construct the likelihood ratio test for

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0.$$

Compare to the Wald test.

Exercise 6 (Testing for a Binomial Proportion)

(**LW** 10.13) Let $X \sim \text{Binomial}(n, p)$. Construct the likelihood ratio test for

$$H_0 : p = p_0 \quad \text{versus} \quad H_1 : p \neq p_0.$$

Compare to the Wald test.

Exercise 7 (Comparing Power)

(**CB** 8.13) Let $X_1, X_2 \sim \text{Uniform}(\theta, \theta + 1)$. Consider two tests for

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta > 0.$$

1. $\phi_1(X_1)$: Reject H_0 is $X_1 > 0.95$.
2. $\phi_2(X_1, X_2)$: Reject H_0 is $X_1 + X_2 > c$.

With these two tests...

- Find the value of c that makes ϕ_2 have the same size as ϕ_1.
- Calculate and plot the power of each test.

Exercise 8 (Using Neyman-Pearson)

(**CB** 8.20) Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x \mid H_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f(x \mid H_1)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = 0.04$. Compute the probability of a Type II Error for this test.

Exercise 9 (Multiple Testing)

(**LW** 10.11) A randomized, double-blind experiment was conducted to assess the effectiveness of several drugs for reducing postoperative nausea. The data are as follows.

Treatment	Number of Patients	Incidence of Nausea
Placebo	80	45
Chlorpromazine	75	26
Dimenhydrinate	85	52
Pentobarbital (100 mg)	67	35
Pentobarbital (150 mg)	85	37

Test each drug versus the placebo at the 5% level with a two-sided test. Report the p-value for each test. Also report Bonferroni corrected p-values.

Exercise 10 (Verifying Size)

(**LW** 10.12) Let $X_1, \ldots, X_n \sim \text{Poisson}(\lambda)$. Let $\lambda_0 > 0$. Find the size α Wald test for

$$H_0 : \lambda = \lambda_0 \quad \text{versus} \quad H_1 : \lambda \neq \lambda_0.$$

Set $\lambda_0 = 1$, $n = 20$, and $\alpha = 0.05$. Simulate from the null distribution and perform the Wald test. Repeat many times and count how often you reject the null. How close is this result to $\alpha = 0.05$.

Exercise 11 (Simulating Power)

Use the setup from Exercise 10 and perform a simulation study that is informative about the power of the test.